Extracting Basic Information from a Paraxial Ray Trace

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Paraxial ray tracing is used to follow a ray through an optical system consisting of a number of surfaces, and the spaces that they bound. Limited to paraxial rays, and first order optics, this method carries inside it the assumption that $sinx \approx x$ for x in radians. This is the first term from the power series for sine, and is accurate within about 10% until about 10°. Ray tracing of this type is governed by a handful of equations. The primes used in these equations indicated the 'next' index (n), height (y), or angle (u). The power equation defines the optical power (commonly in diopters, but can be left in any inverse length unit as long as everything else is kept in those units) for a given surface in terms of the index in front of the surface (n), behind the surface (n'), and the radius of that surface (R). The refraction equation similarly relates the angle inside one space to the angle and height the ray meets that surface bounding that space at. Lastly the transfer equation relates how the ray height changes as it propagates over some reduced thickness (t/n), given a height on the first surface (y), the reduced distance traveled (t'/n') and the angle relative the optical axis (n'u') this equation yields the height at the next surface (y').

Power Equation	$\phi = \frac{n'-n}{R}$
Refraction Equation	$n'u' = nu - y\phi$
Transfer Equation	$y' = y + \left(\frac{t'}{n'}\right)(n'u')$

The reduction of the sine functions by first order approximation means that both the refraction and transfer equations are linear, which allows one to easily fill in a 'brick-chart' style ray tracing sheet fairly quickly, and also becomes useful when it is necessary to scale rays. In this paper I will assume that basic knowledge of the applying these ray tracing equations, and the use of the chart, is understood, and will only detail the explicit extraction of important information.

Element	Method
Effective Focal Length (efl)	Divide the true marginal ray height at the first surface, by the negative of the marginal ray angle in image space.
Entrance Pupil Location	Divide the chief ray height at the first surface by the chief ray angle at that surface. Distance is relative to first surface.
Entrance Pupil Diameter	The full diameter is given by twice the Lagrange invariant, divided by the chief ray angle in object space. $$\frac{2\ \text{Ж}}{n\bar{u}_{obj}}$$
Exit Pupil Location	Divide the chief ray height at the last surface by the chief ray angle at that surface, and multiply by negative one. Distance is relative to last surface.
Exit Pupil Diameter	The full diameter is given by twice the Lagrange invariant, divided by the chief ray angle in image space. $$\frac{2\ \text{Ж}}{n\bar{u}_{img}}$$
Back Focal Distance (BFD)	The distance from the last surface to the image can be found by multiplying the marginal ray height at the last surface by the index of the media in image space, and dividing that quantity by negative one times the marginal ray angle in image space. This is equivalent to using the transfer equation to find the distance for which the marginal ray height is zero.
Principal Planes (P, P^*)	The distance from the first and last surfaces to the principal planes can be found be tracing the marginal ray backwards in image space (do not cross any surfaces, simply extend the ray some distance x in the negative z direction, up to the height it had on the first surface. Note this only works if the marginal ray came in parallel to the optical axis (object at infinity). The distance x is the distance from the last surface to P^*. To find the front principal plane, one must flip the system around, and trace a new true marginal ray, and use the same procedure.
Field of View	The semifield (SFOV) of view is found by simply converting the true chief ray angle in object space into degrees, this value is denoted as a plus-minus, ex. $\pm 5°$. For full field of view (FOV), you need to double that value.
Field Stop	The field stop dimension can be found by multiplying the effective focal length (efl) by the full field of view in radians (FFOV).
Lagrange Invariant (Ж)	The Lagrange invariant does not change as a ray bundle moves through a system. It is easiest to define this at an image or object (when the marginal ray height, y is zero), or at the aperture stop or pupils (where the chief ray height $\bar{y}$ is zero). It is of note, the throughput of a system is related to the square of this value. $$\text{Ж} = n\bar{u}y - nu\bar{y}$$