Plastic Deformation and Failure in Solids

Alexander Miles
AlexanderAshtonMiles@gmail.com

Written: December 30, 2008
Last edit: January 4, 2009

When materials are put under a stress of a given type and magnitude, it will eventually fail. The nature of this failure can be predicted based upon the structure of the material, and the modes of failure fall into two general classes: plastic and brittle. Plastic deformation implies that the object or sample changes in shape, and does not return to its original state when the stress is removed (if it did, that would be considered elastic deformation). Brittle failure occurs when the sample breaks without deforming.

---

1 Plastic Deformation

Plastic deformation is usually only spoken of in the context of metallic and other crystalline solids, as they are generally the only ones that show significant plastic deformation. It occurs when an applied stress is large enough to allow dislocations in the crystal lattice to move, in a process called slip. Imagine the way a caterpillar moves, with a dislocation moving from one end to the other, causing a small movement of the bug. Due to the geometry of each crystal system (BCC, FCC, HCP, and SC), there are several slip planes, that is a plane of atoms that will move under a given resolved stress, and each plane has several slip directions, the directions that the plane may move in. Together, a slip plane and slip direction are referred to as a slip system. The stress required to initiate a slip system (usually under some given conditions, as some slip systems are not "available" until high temperatures have been reached) is denoted σ_y. Note that most metals can be strain or work hardened, that is, after being plastically deformed, the metals then have a higher yield stress, as well as a more brittle behavior. This is due to the fact that deformation introduces defects, which are in themselves an impediment to dislocation movement.

1.1 Peierls-Nabarro Stress

Denoted τ_{p-n}, Peierls-Nabarro Stress quantifies the stress required to move one dislocation from one lattice site to another.

$$\tau_{p-n} = Ge^{\frac{-2\pi W}{b}} \tag{1}$$

To begin to understand this quantity, we need to define a few things. G is the shear modulus of the material in question, W is the dislocation width, or inter-planar spacing between adjacent planes of atoms, and b is the magnitude of the Burger's Vector. The Burger's Vector points in the direction of dislocation movement, and has magnitude equal to the distance the dislocation must move to reach another site. For example, in a close packed direction $|\vec{B}| = 2R_{atomic}$. From this expression we see that dislocations (and thusly plastic deformation) occur more easily when τ_{p-n} is small, implying that close packed directions (smaller b), large inter-planar spacing (bigger W), and smaller shear modulus all favor plastic deformation.

1.2 Slip Systems

Each crystal system (FCC, BCC, HCP, and SC) have their own sets of distinct slip systems, which contribute to plastic behavior. Here are the preferred slip systems in the common crystal systems, given in terms of miller indices indicating a family of planes, ex. $\{000\}$, and a family of directions, ex. $< 000 >$.

Crystal System	Slip Plane	Slip Direction	Number of Slip Systems
FCC	$\{111\}$	$< 1\bar{1}0 >$	12
BCC	$\{110\}$	$< \bar{1}11 >$	12
BCC	$\{211\}$	$< \bar{1}11 >$	12
BCC	$\{321\}$	$< \bar{1}11 >$	24
HCP	$\{0001\}$	$< 11\bar{2}0 >$	3
HCP	$\{10\bar{1}0\}$	$< 11\bar{2}0 >$	3
HCP	$\{10\bar{1}1\}$	$< 11\bar{2}0 >$	6

Geometrically, the slip systems in FCC and BCC intersect quite often, while the systems in HCP are mostly parallel to each other. These intersections allow for cross slip, a process by which a dislocation that encounters and obstacle (often a defect or grain boundary) can switch to a different and intersecting slip system and continue to propagate. This is one of the reasons that metals we deal with day to day display plastic behavior, as they are almost always polycrystalline.

1.3 Schmidt's Equation

Schmidt's equation is used to determine the magnitude of the resolved stress in a given slip system, meaning for one slip plane and one slip direction, for a given uniaxial stress. ϕ is the angle between the applied stress and the normal to the slip plane, and λ is the angle between the applied stress and the slip direction.

$$\sigma_R = \sigma_0 \cos(\phi) \cos(\lambda) \tag{2}$$

It is immediately clear that if the stress is parallel to either, it will necessarily be perpendicular to the other. This means that a force will have the maximum resolved stress when it is applied at a 45° angle from both the plane normal and the slip direction. Each slip system has a critical resolved shear stress (CRSS) that marks the threshold of resolved stress beyond which plastic deformation will occur. If the applied stress becomes very large, and the angle of application such that the resolved shear stress does not exceed the CRSS, brittle failure may occur.