Basic Examples of Magnetism in Physics

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Magnetism is a physical phenomenon closely linked with electricity. Here I will outline the basic laws that govern magnetism in physics, and give a few examples for the purpose of practice.

Equations Overview:

Boit-Savart Law - Magnetic field generated from a single isolated current.

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{s} \times \widehat{r}}{r^2} \tag{1}$$

Ampère's Law - Magnetic field generated from a distribution of constant currents.

$$\oint \vec{B} = \mu_0 I_{enclosed} \tag{2}$$

Solenoid Field - Field inside an ideal infinite solenoid with n turns per meter.

$$|B| = \mu_0 n I \tag{3}$$

Faraday's Law - Voltage induced by a magnetic flux through a coil of N turns.

$$\mathcal{E} = \oint \vec{B} \cdot d\vec{s} = -N \frac{d\Phi_B}{dt} \tag{4}$$

Example 1

Consider two infinite anti-parallel currents, $I_1 = 5A$ and $I_2 = 4A$ separated linearly by a distance $D = 0.5m$, with I_2 right of I_1. Assuming $I_1 > I_2$, at what distance away from I_2 is the magnetic field exactly zero?

We begin with the Boit-Savart Law, as that describes the field from a single current.

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{s} \times \hat{r}}{r^2} \tag{1}$$

If we use the right hand rule, we can determine the direction of the fields are the same between the two currents, and completely opposing to the right of I_2, so we must find the distance at which their magnitudes are equal and opposite. For infinite wires the Boit-Savart Law reduces to the following, where ρ is the distance from the wire to the point of interest.

$$|\vec{B}| = \frac{\mu_0 I}{2\pi\rho} \tag{2}$$

We know that the field from the two wires must total to zero and their directions are opposite, so we can use x as the distance from I_2 to this point, making $D + x$ the distance from I_1.

$$0 = |\vec{B}_1| - |\vec{B}_2| = \frac{\mu_0 I_1}{2\pi(D + x)} - \frac{\mu_0 I_2}{2\pi x} \tag{3}$$

Reorganizing

$$\frac{\mu_0 I_1}{2\pi(D + x)} = \frac{\mu_0 I_2}{2\pi x} \tag{4}$$

$$\frac{I_1}{(D + x)} = \frac{I_2}{x} \tag{5}$$

Inserting known values and cross-multiplying.

$$5x = 4(0.5 + x) \tag{6}$$

$$x = 2 \tag{7}$$

At the point two meters right of I_2 there is no magnetic field at all.

Example 2

Given a loop of wire, 0.1m in radius, made of copper ($\rho = 1.7 \times 10^{-8} \Omega m^{-1}$) with wire-radius of 0.001m. Find the induced current as a uniform upward (relative to the loop) magnetic field increases linearly from 0 Tesla to 1.5 Tesla in 10 seconds.

We know that Faraday's Law gives the induced voltage in the loop, so we begin there.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \tag{1}$$

We have only one loop, so $N = 1$. Secondly, we know that the field changes linearly, so the slope is constant.

$$\frac{d\Phi_B}{dt} = A\frac{dB}{dt} = A\frac{\Delta B}{\Delta t} \tag{2}$$

$$\frac{d\Phi_B}{dt} = \pi(0.1)^2\frac{1.5}{10} = 0.0015\pi \tag{3}$$

To get current from voltage, we make the assumption that the loop is ohmic, and obeys the following equation.

$$\frac{V}{R} = I \tag{4}$$

To extract resistance from the resistivity (ρ), we use the following.

$$R = \frac{\rho L}{A} \tag{5}$$

Now we can substitute the given resistivity, and the computed length (circumference of the loop) and cross-sectional area of the wire.

$$R = \frac{1.7 \times 10^{-8} \times 0.2\pi}{\pi(0.001)^2} = 0.0034\Omega \tag{6}$$

Now we utilize the ohmic relation to find the current.

$$\frac{0.0015\pi V}{0.0034\Omega} = 1.386A \tag{7}$$

Example 3

A disc of radius R has a total charge of Q spread evenly over it's surface. It is rotated at an angular velocity ω. Determine the magnetic field at the center of the disc.

We start by recognizing that each 'ring' of charge on the disc is rotating at a different linear velocity, and that we can break the disc into these rings and find the field from each ring. We utilize polar coordinates and the corresponding area element.

$$I = \frac{dQ}{dt} \tag{1}$$

$$dQ = \sigma \rho d\rho d\phi \tag{2}$$

$$dt = \frac{2\pi}{\omega} \tag{3}$$

We can use the following for the field at the center of a single loop.

$$|\vec{B}| = \frac{\mu_0 I}{2\pi\rho} \tag{4}$$

Substitute in the expression for current.

$$|\vec{B}| = \frac{\mu_0}{2} \int\int \frac{\omega\sigma\rho d\rho d\phi}{2\pi\rho} \tag{5}$$

We need to integrate to find the current, I. The limits allow us to integrate over the entire disc. At this point we can also substitute in for sigma, the charge density, using the condition that the distribution is even.

$$\sigma = \frac{Q}{\pi R^2} \tag{6}$$

$$|\vec{B}| = \frac{\mu_0}{2} \int_0^{2\pi} \int_0^R \frac{\omega Q d\rho d\phi}{2\pi^2 R^2} \tag{7}$$

Pull out the constants.

$$|\vec{B}| = \frac{\mu_0 \omega Q}{4\pi^2 R^2} \int_0^{2\pi} \int_0^R d\rho d\phi \tag{8}$$

$$|\vec{B}| = \frac{\mu_0 \omega Q}{4\pi^2 R^2} 2\pi \int_0^R d\rho \tag{9}$$

$$|\vec{B}| = \frac{\mu_0 \omega Q}{2\pi R^2} R \tag{10}$$

$$|\vec{B}| = \frac{\mu_0 \omega Q}{2\pi R} \tag{11}$$